

Name: Key

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- You have one hour to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Your final answers **must start with** $f'(x) =$, $\frac{dy}{dx} =$, or similar.
- **Draw a box around your final answer.**

1. [12 points] Compute the derivatives of the following functions.

a. $f(x) = x \sin(x)$

$$f'(x) = \sin x + x \cos x$$

b. $f(x) = \frac{1}{4x} + \sqrt{4x} = \frac{1}{4}x^{-1} + 2x^{\frac{1}{2}}$

$$f'(x) = -\frac{1}{4}x^{-2} + x^{-\frac{1}{2}}$$

c. $f(x) = \frac{\sin(x)}{\cos(x)} = \tan x$

$$f'(x) = \sec^2 x$$

d. $f(x) = e^{(x^3 - 4x^2 + 7)}$

$$f'(x) = e^{x^3 - 4x^2 + 7} (3x^2 - 8x)$$

e. $f(x) = \frac{\cos(x/2)}{2x^4}$

$$f'(x) = \frac{-\sin\left(\frac{x}{2}\right) \cdot \frac{1}{2} \cdot 2x^4 - \cos\left(\frac{x}{2}\right) \cdot 8x^3}{(2x^4)^2}$$

f. $f(x) = \ln(\sec x + \tan x)$

$$f'(x) = \frac{1}{\sec x + \tan x} (\sec x \tan x + \sec^2 x)$$

g. $f(x) = \sqrt{x + \ln(3x)}$

$$f'(x) = \frac{1}{2} (x + \ln(3x))^{-\frac{1}{2}} \cdot \left(1 + \frac{1}{3x} \cdot 3\right)$$

h. $f(x) = \frac{x \ln(x)}{\ln 3}$

$$f'(x) = \frac{1}{\ln 3} \left(\ln x + x \cdot \frac{1}{x} \right)$$

i. $y = \pi \left(\frac{1+x}{2} \right)^4$

$$\frac{dy}{dx} = 4\pi \left(\frac{1+x}{2} \right)^3 \cdot \left(\frac{1}{2} \right)$$

j. $f(x) = (\cos(x^2 + e^2))^5$

$$f'(x) = 5(\cos(x^2 + e^2))^4 \cdot (-\sin(x^2 + e^2)) \cdot 2x$$

k. $f(x) = \tan^{-1} x$

$$f'(x) = \frac{1}{1+x^2}$$

l. Find $\frac{dy}{dx}$ for $2x + y = y \sin(x)$. You must solve for $\frac{dy}{dx}$.

$$2 + \frac{dy}{dx} = \frac{dy}{dx} \cdot \sin x + y \cos x$$

$$\Rightarrow \frac{dy}{dx} - \sin x \cdot \frac{dy}{dx} = y \cos x - 2$$

$$\Rightarrow \frac{dy}{dx} = \frac{y \cos x - 2}{1 - \sin x}$$